The Planck scale: relativity meets quantum mechanics meets gravity.

According to Special Relativity's time dilation and length contraction, two observers with relative motion are condemned to eternal disagreement about times and lengths. Or are they?

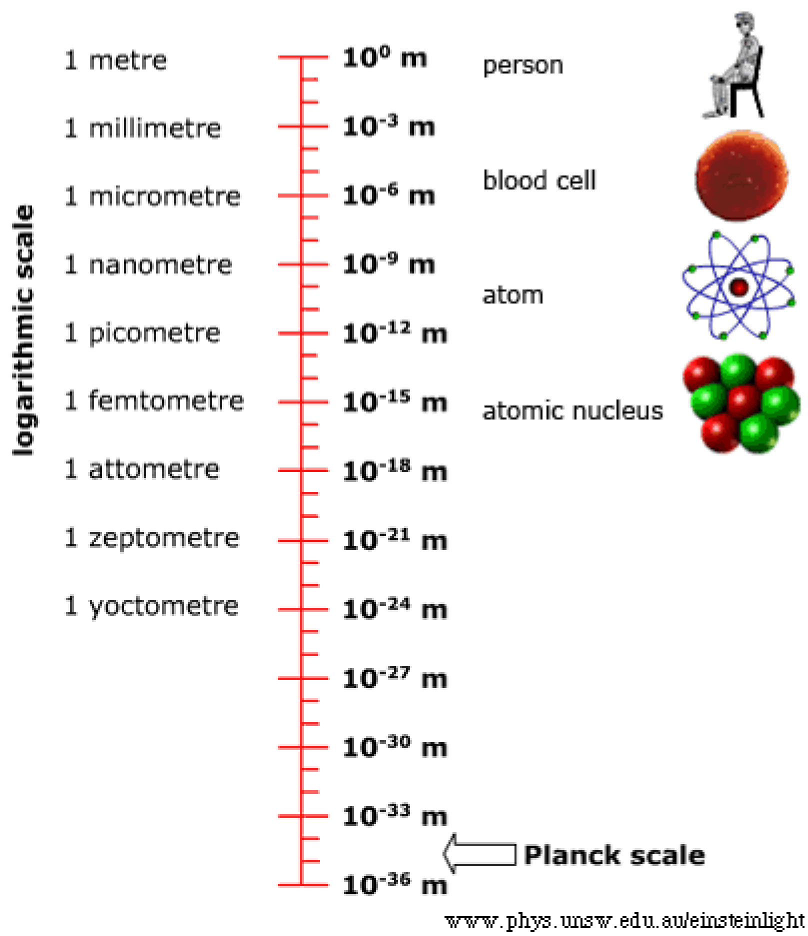
They will disagree over the arbitrary times measured in seconds and lengths measured in metres or whatever units. They would also disagree over clock ticks, atomic vibrations and light wavelengths. But what if there were some absolute times and lengths, determined by the very nature of the universe? And what if these quantities could be measured in different frames of reference. Wouldn't the observers have to agree on those having the same value, if they agree on the same laws of physics?

Well, there is an abolute length and an absolute time. It's possible that we'll never be able to measure them, so any threat posed to relativity is distant and perhaps even hypothetical. It is interesting to see whether and how these quantities might be included in relativity.

The **Planck length** L_P is defined by taking the constants of nature and combining them in such a way that their units combine to give a length. Planck's constant, h, has units of Joule seconds. (These days, physicists usually work with $h/2\pi$, and so shall we.) Cavendish's constant G (the constant of gravitation) has units of $N.m^2kg^{-2}$. And the speed of light c has units of $m.s^{-1}$. The combination that works is:

$$L_P = (hG/2\pi c^3)^{1/2} .$$

The Planck length is 1.6×10^{-35} metres. (That's



0.0000000000000000000000000000000016 meters.) To give you an idea, let's compare it with the size of an atom, which is already about 100,000 times smaller than anything you can see with your unaided eye (an atom size is about 0.0000000001 meters). Suppose that you measured the diameter of an atom in Planck lengths, and that you counted off one Planck length per second. To measure the atomic diameter in Planck lengths would take you 10,000,000 times the current age of the universe. (After which time, for Joyce fans, the fires of hell may well have begun to begin to burn.)

Divide the minuscule Planck length by the speed of light (which is pretty big) and you get a really tiny unit of time, the **Planck time**, t_P, which is:

$$t_P = (hG/2\pi c^5)^{1/2} .$$

The Planck time is 5.4×10^{-44} seconds. (I'm wearing out the zero key on this keyboard, so I shan't write it out in full but you get the idea: it's brief.)

There is also a Planck mass, which is $(ch/2\pi G)^{1/2} = 22$ µg. This doesn't sound very much, until you think of a fundamental particle with that mass. Or until you convert it into energy by multiplying by c^2 to get $2.0\ 10^9$ joules or 1.2×10^{28} eV. Yes, 2 billion joules all concentrated in one atomic particle.

Anyhow, the Planck length and time are very small, but they are results solely of the values that appear naturally in our physical laws. So according to the principle of Special Relativity, it seems that different observers should observe them to be the same. So what about time dilation and length contraction? If these lengths and times are observable as physical lengths and intervals in moving frames, it appears that we shall need to modify Special Relativity to include them. One theory that does so is called Doubly Special Relativity, suggested in 2002 by Giovanni Amelino-Camelia.

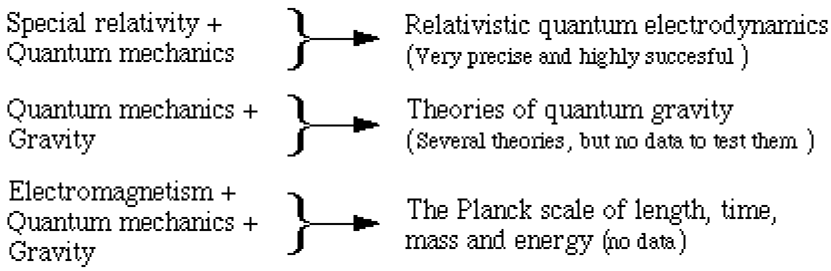
Before we get too excited, we should point out just how far beyond current experimental technology these effects are. Particle accelerators are described by the energies that they can produce, and the latest generation produces energies of TeV, or 10^{12} eV. The Planck energy is 1.2×10^{31} eV. We are short by a factor of 10^{19}. Which is a good thing.

Further, it's not clear (to this author, at least) what it would mean to measure these lengths and times in or from different frames of reference. As we'll see below, on the Planck scale, time and space no longer have their ordinary, macroscopic meaning and so naïve applications of relativity are probably inappropriate.

Quantum mechanics, gravity and relativity

So, where do these quantities come from? The speed of light c is the natural unit that relates time and space. G is the constant of gravity, and h is the constant of quantum mechanics. So the Planck scale defines the meeting point of gravity, quantum mechanics, time and space. Currently, we don't know much about this interaction, because gravity is so feeble that its influence on things as small as quantum systems is small.

Special Relativity and quantum mechanics work very well together. Relativistic quantum electrodynamic is a spectacularly accurate theory. Richard Feynman once described how accurate it was by saying: if you asked me how far it was to the moon and I said "do you mean from my head or from my feet?" That accurate.



Quantum mechanics and gravity (whether Newton's theory of gravity or Einstein's theory of General Relativity) do not fit so neatly together. The problem can be put in several different ways, but I favour this one. From our discussion of virtual particles (Why there would be no chemistry without relativity), we saw that virtual particles could be larger (ie more massive) if their lifetime and range were smaller.

Now both Newton's and Einstein's gravity predict that enough mass in a small enough space can produce a black hole: a region with a gravitational field so strong that its escape velocity is c. When we put the two ideas together, we find that there is a scale small enough for virtual black holes to exist. This is the Planck scale. On this scale, all of the weird, singular behaviour associated with black holes asserts itself. Space and time as continuous entities cease to have meanings when discussing distances of 10^{-35} metres and times of 10^{-44} seconds. So relativity, a theory of space and time based on a continuum, must run into serious difficulties.

Which is perhaps not surprising: the Planck scale is a very, very long extrapolation from our current knowledge.

On this topic, we have so little direct knowledge that there are few hints to guide the development of theories, and even fewer constraints upon those theories. Consequently, there are several different families of theories that aim to produce a consistent theory of quantum gravity. Usually they include a larger number of spatial dimensions, not all of which are macroscopic*. At the moment, however interesting they be, these theories are speculative. Perhaps one of them will turn out to be a good, useful theory, and the others will fall. At the moment, we cannot put them to the test.

Today, we remember Democritus for speculating on the existence of atoms and Aristachos for proposing that the Earth went around the sun. The ancient Greek philosophers proposed so many ideas that it is perhaps not too surprising that some of them turned out to be consistent with facts discovered much later.

---

* How can we have more than three spatial dimensions? Surely Gauss' law (for electricity, magnetism and gravity) shows that we live in a locally flat geometry with three spatial dimensions?

Yes, experiments to test Gauss' law either directly or indirectly do show that our geometry is both pretty flat and three dimensional *on the scale of the experiments conducted*. If, however, the universe were closed in all but three of the spatial dimensions, and if in the closed dimensions the radius of the universe were much smaller than the size of measurements, then Gauss' law would apply only to the three large dimensions. Consequently electricity, magnetism and gravity would be inverse square laws on the scales that are experimentally accessible. The theories that use extra dimensions then have the possibility to use such things as standing waves on circumference of the universe in the closed dimensions.

PHYSICS@UNSW

scienceUNSW